order is reduced to 3.

The mutual influence of the narrow cavities is traced only in the lower harmonics $p = 0, \pm 1$ of the Fourier-series expansions; starting with $p \geqslant 2$ it is negligible. The presence of adjacent cavities exerts no influence on the determination of the functions

$$Q_j(a, 0, \alpha),\quad P_j(a, 0, \alpha),\quad \tau_j(a, \pm 1, \alpha),\quad Q_j(a, \pm p, \alpha),\quad p \geqslant 2$$

These functions introduce a contribution of the order of a^2 and higher to the wave field. The functions $\tau_j(a, 0, \alpha), Q_j^1, Q_j^2$ depend on the mutual location of the cavities and the stresses on their walls. The wave-field component due to these functions is of the order of $1/\ln a$. Taking account of (2.5) and the above, we arrive at the conclusion that the wave field of the problem is determined by the functions $\tau_j(a, 0, \alpha), Q_j^1, Q_j^2$. The displacement field of the initial problem is determined by (3.1), (2.2), (1.1); it is sufficient to set $p, m = 0, \pm 1$ in (2.2)

In the case $N = 1$, relationships (3.1) take the simplest form

$$u^\circ(r, \varphi, z) = \frac{1}{\ln a}\int\limits_\sigma e^{i\alpha z}\left\{-\frac{i\alpha}{\varkappa_2^2}y_4(a, z)W^*(a, 0, a) + \right.$$

$$\left.\frac{K_{11}^1(r, 1, \alpha, a)}{\varkappa_1^2 + \varkappa_2^2}\left[e^{i\varphi}U^*(a, 1, \alpha) + e^{-i\varphi}U(a, -1, \alpha)\right]\right\}d\alpha + u^*(r, \varphi, z)$$

$$v^\circ(r, \varphi, z) = \frac{1}{\ln a\,(\varkappa_1^2 + \varkappa_2^2)}\int\limits_\sigma e^{i\alpha z}K_{21}^1(r, 1, \alpha, a)\left[e^{i\varphi}U^*(a, 1, \alpha) + e^{-i\varphi}U^*(a, -1, \alpha)\right]d\alpha + v^*(r, \varphi, z)$$

$$w^\circ(r, \varphi, z) = -\frac{1}{\ln a}\int\limits_\sigma \varkappa_2^{-2}e^{i\alpha z}\left\{y_1(\alpha, r)W^*(a, 0, \alpha) + \right.$$

$$\left.\frac{i\alpha}{\varkappa_1^2 + \varkappa_2^2}y_2(\alpha, r)\left[e^{i\varphi}U^*(a, 1, \alpha) + e^{-i\varphi}U^*(a, -1, \alpha)\right]\right\}da + w^*(r, \varphi, z)$$

It should be noted that near the side surface of the cavities the displacement field components caused by the influence of the adjacent cavities and the cavity itself are related as $1 : \ln a$.

A system of cavities of small radius a that perforates an elastic space causes a perturbation of the order of $1/\ln a$ in the wave field of an elastic medium. This contribution depends on both the number of cavities in the system and on their arrangement.

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THE POSSIBILITY OF IRREVERSIBLE QUASISTATIC PROCESSES IN A MACROSYSTEM*

A.A. VAKULENKO

For a fairly long time only such systems for which the running macrostate of the system is practically independent of the preceding history of change in the external parameters for quasistatic * (The definition of a quasistatic process used here corresponds to the standard definition (for instance, /1–5/), while the reversibility of the process is understood in the narrow sense /5/.) processes, have been considered in thermodynamics. The absence of a "memory" is characteristic for quasistatic processes in gases and ordinary liquids, as for reversible processes in any system.

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> But the quasistatics (equilibrium) of the process does not always denote the absence of a "memory" and, thereby the nearness of the process to reversible. From the formal viewpoint, this is associated with the fact that not every differential form is an exact differential (integrating) form.

Let us consider some fairly simple thermodynamic system for which the complete set of external parameters forms a finite set of variable quantities. We denote these parameters by $a_1, a_2, \ldots, a_n$ and we agree that the thermostat temperature is also in their number for the system in a thermostat. Then the completeness of this system of external parameters a_i means that always, when each of them is a given function of time in the interval $-\infty < \tau < t$, all the internal parameters ξ_α of the thermodynamic system under consideration will also be defined at time t. Let the functions $a_i = a_i(t)$ be able to be given such that for each $t \in (-\infty, \infty)$ and each of the parameters ξ_α a relationship of the following form holds:

$$\frac{d\xi_\alpha}{dt} = \sum_{k=1}^{n} C_{\alpha k} \frac{da_k}{dt} \tag{1}$$

where $C_{\alpha k}$ are certain functions of the parameters ξ_α, a_k and, possibly, other characteristics of the process trajectory in the system state space. For suitable boundedness conditions on the functions $C_{\alpha k}$ it follows from (1) that when all the external parameters are fixed at a definite time $da_k/dt = 0$ for $t \geqslant t_0$ (and for all $k = 1, 2, \ldots, n$) the system state will also be invariant for $t \geqslant t_0$. This means that each of the system states in the process is an equilibrium state. In order for the process to be not only in quilibrium but also reversible, it is necessary that for a fixed initial state of the system the running values of all its state parameters should depend only on the running values of the external parameters: $\xi_\alpha = \xi_\alpha(a_1, a_2, \ldots, a_n)$, and this will result from (1) only in the case when the functions $C_{\alpha k}$ satisfy a definite kind of additional condition. Thus, if the coefficients $C_{\alpha k}$ in (1) are differentiable functions only of $a_1, a_2, \ldots, a_n$, the forms (1) will be exact differentials of certain functions of these same arguments except under the condition that $\partial C_{\alpha k}/\partial a_l = \partial C_{\alpha l}/\partial a_k$ for all $k, l = 1, 2, \ldots, n$.

Caratheodory turned the attention of specialists at the beginning of the century to the fact that its reversibility does not generally follow from the equilibrium of a process /1/: "It is completely admissible, albeit physically, that there are substances in nature which cannot possibly be considered a component part of any simple system". Such there would be, for instance, if the internal friction of the substance under consideration, which is a function of the strain rate, did not tend to zero for quasistatic changes in state. Indeed, the forces generating the work would not then be comparable to the forces assuring equilibrium; it should be impossible to represent the external work A as a "Pfaffian form" Keep in mind that the differential form

$$dA = \sum_i P_i(a_k)\, da_i$$

governing the elementary work of the external forces cannot be linear but only a homogeneous function of first degree in da_i, while its coefficients are homogeneous functions of zero degree in da_i; particularly of the form

$$dA = \sum_i P_i\left(a_k, \frac{a_k^{\cdot}}{|a_k^{\cdot}|}\right) da_i$$

In a similar manner, the form dA is related to changes in the external parameters in the case, say, of a system represented by a load on a plane subjected to dry friction forces. L.I. Sedov /2/ discussed this example in detail /2/. Also almost all of the models known now for an elastoplastic complex medium /3/ might be an example of process equilibrium in which its reversibility is not definitely implied. The need to distinguish equilibrium and reversible processes was noted also in other work of L.I. Sedov.

Nevertheless, the assertion concerning the reversibility of any quasistatic process can be encountered in not only old but even many modern books and papers /4–6/. The validity of such an identification is ordinarily considered evident; however, attempts to give it a foundation are also known. Here the so-called zeroth (or general) principle of thermodynamics, the postulate that, with all external conditions fixed, a system will reach a state of thermodynamic equilibrijm in time, is always the starting point in some form or other. It is asserted that this state is defined completely by fixed values of the external parameters /4, 5/ from which it follows, in turn, that in an equilibrium process with given initial state of the system, its running state will depend only on the running values of the external parameters; therefore, the process is reversible.

However, it is easy to see that in this reasoning, it is implicitly assumed, in addition to the "zeroth" principle of thermodynamics, that the system state established under invariant external conditions is independent of how the external parameters changed prior to being fixed. For systems being considered in thermodynamics during its establishment, such an assumption has a certain foundation. In a gas or liquid with sufficiently low viscosity, the particle thermal motion indubitably succeeds during the approach to a state of thermodynamic (statistical) equilibrium in "erasing" all that might remain of the states in the equilibrium state prior to the fixed external conditions. But at this time there is no doubt at all that the situation may be different with processes in solids.

For instance, consider the deformation process of a metal single or polycrystal.In the usual laboratory tests, the body ("specimen") is under thermostat conditions, where the appropriate macrocharacteristics of the body geometry (the macrodeformation tensor components) play the part, together with the thermostat temperature, of the external parameters. As slowly as these geometric external parameters seem to change, the time inevitably arrives during the monotonic course of the process, when dislocations start to move in the crystal lattice and specify the appearance and development of plastic deformation. If all the external parameters are again fixed and maintained invariant at a certain time in the course of this process, then the microdefect displacement also ceases with time, a new equilibrium distribution is reached in the body (and moreover, is generally reached considerably more rapidly when the body possesses a stable structure in the initial state and the changes in the external parameters prior to their being fixed completely are slow enough, in which case the relaxation times are measured in fractions of a second /7, 8/). As the results of experiments show, this new equilibrium state of the plastically deformed specimen depends substantially on the process trajectory in the space of the external parameters. The study of this dependence is substantially one of the fundamental problems of plasticity theory, and many experimental results have been obtained in this direction /9/.

Other examples of the processes in solids are also known for which no matter how slow the changes in the external parameters by which they were caused, an essential "memory" is characteristic and, despite their slowness, such processes are irreversible. It is asserted in the comments of D.V. Sivukhin /5/ to the book /4/ that it is quite impossible to consider these processes as equilibrium processes.

Let us briefly examine the arguments in /5/ on whose basis this deduction is made.

1) It is asserted that in some of these processes (for instance, in the magnetization of a ferromagnetic material) the system states are not "ture" equilibrium states but metastable, i.e., when perturbations are imposed the system does not return to the initial state when they are attenuated, but finds itself in another equilibrium state.

We note that such a situation is possible in any system for which not one equilibrium state corresponds absolutely to given external parameters. But until now systems of this type have been considered in thermodynamics, it is sufficient to recall the classical thermodynamic theory of phase transitions, in conformity with which, the possibility of a phase transition for given external conditions will again denote the ambiguity of the equilibrium state under these condtions. The fundamental situations of thermodynamics, including even the "zeroth" principle, do not regulate the uniqueness of the equilibrium achievable by a system under invariant external conditions, and hence there is no foundation for complicating the definition of an equilibrium process by an additional requirement regarding the stability of the equilibrium in such a process.

2) It is asserted that the non-equilibrium of irreversible processes can be masked by the slowness of the approach to equilibrium: for if the process is slow compared with the relaxation time, non-equilibrium "does not succeed" in manifesting itself and the process appears to be in equilibrium, although such is not actually the case.

The validity of this assertion is indisputable but it is more often not in favour of the fundamental thesis of /5/, but of the "validity" of the concept of an equilibrium irreversible process. Indeed, in constructing a rational theory, something must always be neglected, and when the time during which the system state changes noticeably in the process being studied is many times less than the relaxation time, it is natural to neglect relaxation first in describing this process, i.e., to consider the process in equilibrium.

Moreover, the question arises regarding processes during which noticeable changes in the system state occur in times commensurate with the relaxation time or even substantially exceeding it. The process already mentioned of the fairly slow defomration of a solid can be an example.

3) It is asserted that such processes as magnetization of a ferromagnetic material and plastic deformation processes are still always non-equilibrium because changes in the external parameters with any slowness are associated with the jump-like changes in the internal parameters.

4) It is asserted that even a slow displacement of a load along a plane under the effect of dry friction cannot be an example of an irreversible equilibrium process because under real conditions it always occurs at a finite rate. The contradictory opinion on these examples of other authors is explained in /5/ by "absolutization of idealized models".

As regards the example with dry friction, it must be kept in mind that in macroscopic tests, completely sharp non-zero lower bound of the possible velocities is not detected; this boundary turns out to be blurred, and the greater it is, the more accurate the measurement. In general, the remark about idealized models indeed indicates the main source of objections because all the arguments in /5/ reduce in the long run to the fact that in order to avoid "absolutization" it would seem to be necessary to consider the models always with relaxation, no matter how small or slow it might be, and always to take into account the "jumplikeness" of the changes in the microstructure during plastic deformation of a crystalline body, etc.

In this connection, we recall that science always deals with models. Approximate models are often constructed by discarding secondary effects in "more exact" models (the transfer from three-dimensional elasticity theory to the two-dimensional theory of shells, say). However, such a method is not always possible: for a long time, say, models of a statistical ensemble were constructed in statistical mechanics not only without taking account of quantum effects but also without a representation, generally, regarding the existence of such effects.

The model always has limits of applicability, a case can always be indicated in practice when it is not true. Moreover, reality never does agree exactly with a model, even if the model expresses fundamental laws. For instance, in practice there are no conditions during which Newton's first law would be satisfied exactly.

As in every science, thermodynamics deals with models and only with them. For example, it is impossible to consider an ideal liquid as a model while a viscous liquid is the reality itself. And both are idealized models of a different degree of detail, and with different areas of application. Even the fundamental concepts of thermodynamics, adiabatic insulation or equilibrium, for instance, are strongly idealized representations: adiabatic insulation assumes the complete absence of heat conduction and radiation while the equilibrium of a real system is always only apparent (heavy particles dissociate into lighter ones, internal fields fluctuate, etc.).

As for any other models, the construction of models with irreversible quasistatic processes is associated with idealization. This idealization makes theory rational and corresponding to test sin its area in a completely acceptable manner. In no way does it exceed other idealizations which became classical and habitual long ago, in the power of the assumptions utilized. This can be seen by an analysis of any of the processes of the type under consideration.

Thus, at the microlevel the plastic defomration process is actually related to "jumps": dislocations overcome a different kind of obstacle by being displaced in the crystal, the "Peierls barriers", the impurity particles, and the dislocations of the "forest", where passage through the obstacle is ordinarily associated with particle excitation in a certain neighbourhood. But the dimensions of these obstacles are measured in tenths of Angstroms (the "Peierls barrier" is not usually overcome all at once), and for sufficiently slow deformation of the body a small part of the total number of dislocations taking part in the process always passes simultaneously through the obstacle. Consequently, the excitations of the atomic particles associated with these passages are actually localized in microscopic domains distributed randomly in the body, at large distances from each other, as compared with their dimensions, and in practice are not felt at the macrostates of the body.

This also follows from the well-known Orowan formula that relates the macrostrain rate to the mean velocity of the dislocations in the specimen at a given time during simple tension of pure shear: $d\varepsilon/dt = \rho_* b v_*$ (ρ_* is the density of the moving dislocations, and b is the modulus of their Burgers vector). An experiment can always be performed such that $d\varepsilon/dt \simeq 0$ at each instant of the process and to any accuracy. Then $v_* \simeq 0$ also, and this means that the velocity jump of the dislocation element is always accompanied by the same, but opposite in sign, jump in the velocity of the other elements in the dislocation ensemble (other elements of the same dislocation, possibly). Such "mutually cancelling" jumps cannot communicate a resultant momentum to the specimen, and in substance, they are detected in tests only when the macrostrain rate is sufficiently small for a given measurement accuracy. But even in these cases they appear as small jumps in the values of a random function, i.e., as typical fluctuations.

Snaps and decrepitations in a specimen will sometimes be distinguishable during the process in tests on single crystals, but they are not related to elementary acts of plastic deformation but to the formation of "twins", "fault bands", etc., comparatively large lattice defects because of the difficulties, on the other hand, or ordinary plastic deformation mechanisms in definite cases /8/. Such defects are not formed in the slow deformation of metals with their typical cubic crystalline lattices, and only the noted small jumps, localized randomly in the body and in time, occur at the microlevel. There are no reasons for considering that neglecting these fluctuations in the description of plastic deformation processes is a greater idealization than is imposed in the concept of thermodynamic equilibrium itself.

This idealization also does not contradict the laws of thermodynamics, as well as generally nothing more than a certain tradition. However, there is also another tradition, dating back to Caratheodory and other scientists (thus, way back in 1871 Saint Venant clearly mentioned the equilibrium of plastic deformation processes under certain conditions /9/). Now an entire set of specific macrosystem models is known in which quasistatic irreversible processes occur. These models are included in the exposition of thermodynamics at the modern level and are utilized extensively to solve different theoretical and applied problems.

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